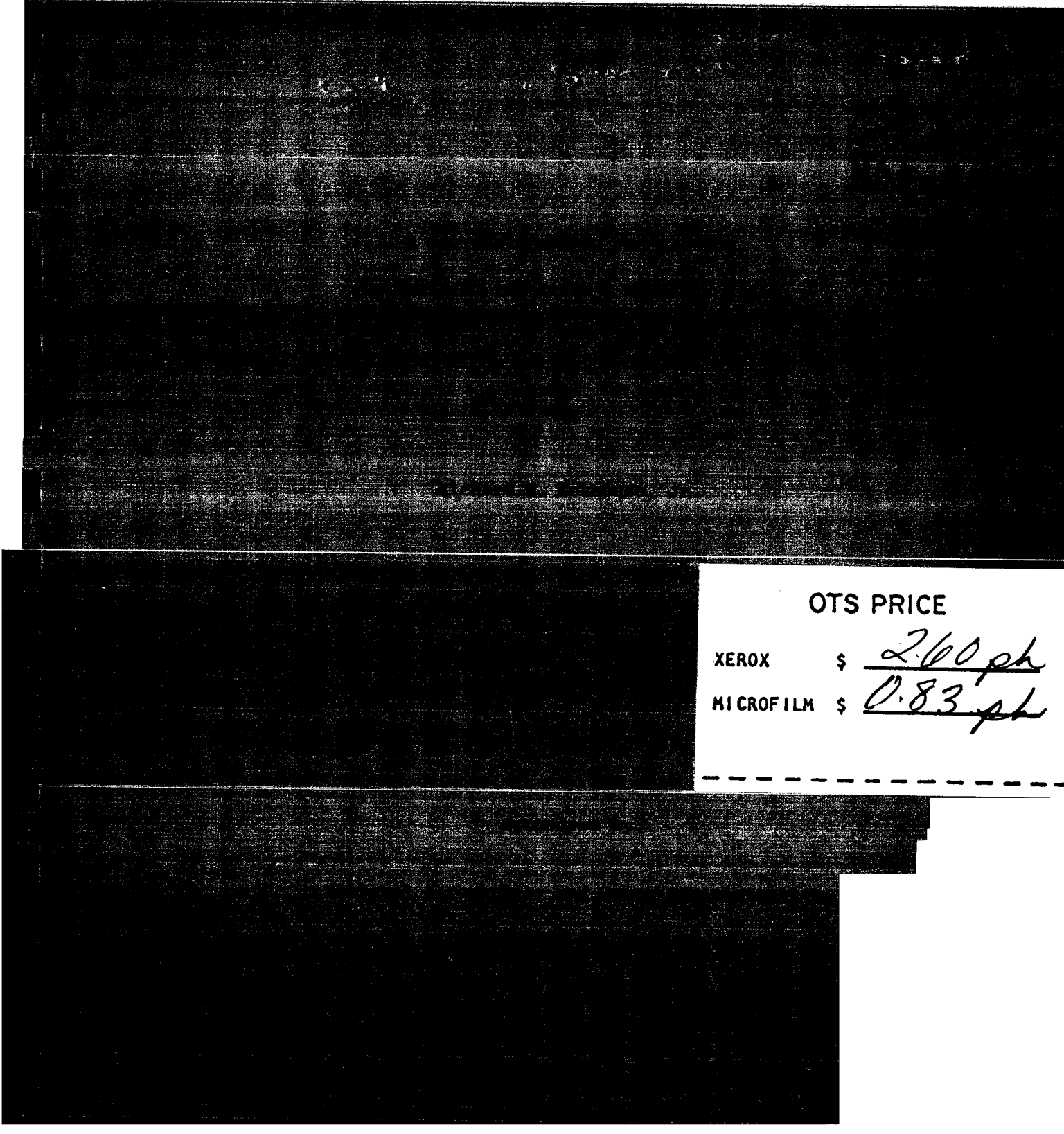

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A FORTRAN COMPUTER PROGRAM FOR DETERMINING THE EARTH'S MAGNETIC FIELD

by

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1. Introduction

Most work in the ionosphere at one time or another involves the use of the earth's magnetic field. The following computer program to compute the earth's magnetic field was specifically designed for Faraday rotation studies. However, the program is rather general in nature and therefore applicable to almost any type of experimental studies.

Using the method given by Jones and Melotte (1953) and the earlier computer program of Yeh and Gonzalez (1960), a more up-to-date program in FORTRAN IV is presented. The spherical harmonic expansion uses the 48 coefficients of Jensen and Cain (1962) which results in terms of sixth order and sixth degree. These equations require very little computer time and make the use of a computer very worth while.

The program itself was designed as a general subroutine for the IBM 7094 computer. Because the program is written in FORTRAN, it should be a simple matter to adapt the program to other FORTRAN compliers or even ALGOL and other compiling systems.

2. Mathematical Expressions

The expressions used in the program are from Jones and Melotte. The Legendre and associated Legendre polynomials are from Morse and Feshback (1953). The magnetic vector potential in spherical harmonics is given by:

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} H_n^m(\mu) \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) \qquad (1)$$

where the following symbols are used:

V = magnetic potential

a = radius of the earth

r = distance from magnetic field point to the earth's center

θ = geographical colatitude

ϕ = geographical longitude

g_n^m and h_n^m = Gaussian coefficients

μ = $\cos \theta$

$$H_n^m(\mu) = \frac{2^n\, n!\, (n-m)!}{2n!}\, P_n^m(\mu) \tag{2}$$

$$P_n^m(\mu) = (1-\mu^2)^{m/2} \frac{d^m}{d\mu^m} P_n(\mu) \tag{3}$$

= the associated Legendre polynomial

The X, Y, Z orthogonal components representing the southward, eastward and upward components of the field are:

$$X = -\frac{1}{\sin\theta} \sum_{n=0}^{6} \left(\frac{a}{r}\right)^{n+2} C_{\theta n} = H_\theta \tag{4}$$

$$Y = \frac{1}{\sin\theta} \sum_{n=0}^{6} \left(\frac{a}{r}\right)^{n+2} C_{\phi n} = H_\phi \tag{5}$$

$$Z = -\sum_{n=0}^{6} (n+1) \left(\frac{a}{r}\right)^{n+2} C_{rn} = H_r \tag{6}$$

where

$$C_{\theta n} = \sum_{m=0}^{n} K_n^m \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) \tag{7}$$

$$C_{\phi n} = \sum_{m=0}^{n} m H_n^m \left(-g_n^m \sin m\phi + h_n^m \cos m\phi\right) \tag{8}$$

$$C_{rn} = \sum_{m=0}^{n} H_n^m \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) \tag{9}$$

$$K_n^m = -\sin\theta \frac{dH_n^m}{d\theta} \tag{10}$$

It should be noted that although the summation indices begin with zero, the Gaussian coefficients for n and m equal to zero do not exist. Since free magnetic charges do not exist, as compared to free electric charges, the monopole term can not exist and in actuality, the expansion begins instead with the dipole term.

One further point is the obvious fact that the expansion is written for a spherical earth.

3. Method of Computation

The flow chart, figure 1, gives the steps used in computing the earth's magnetic field by using equations 4 through 23. Figures four through six explain the spherical trigonometry used.

Flow Chart

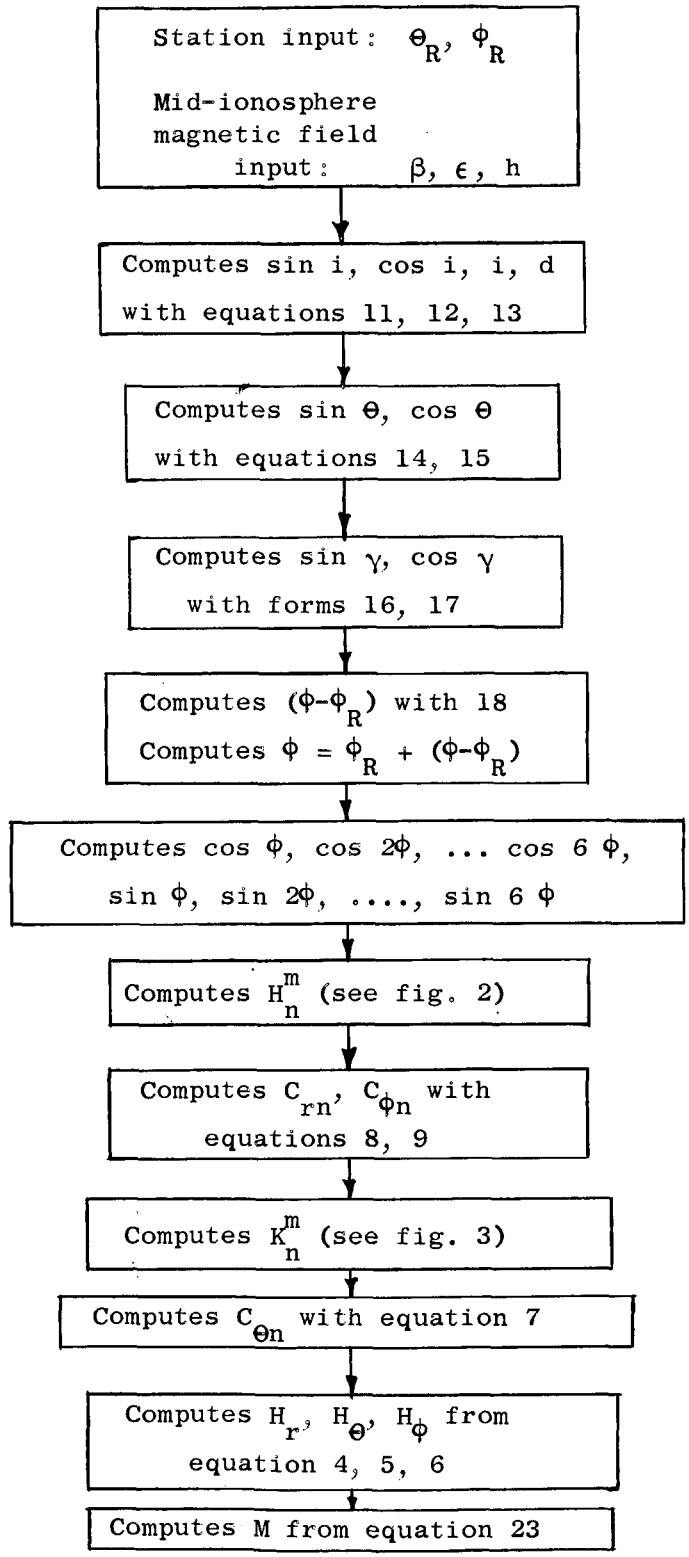

Station input: Θ_R, ϕ_R
Mid-ionosphere magnetic field input: β, ϵ, h
Computes sin i, cos i, i, d with equations 11, 12, 13
Computes sin Θ, cos Θ with equations 14, 15
Computes sin γ, cos γ with forms 16, 17
Computes $(\phi-\phi_R)$ with 18
Computes $\phi = \phi_R + (\phi-\phi_R)$
Computes cos ϕ, cos 2ϕ, ... cos 6ϕ, sin ϕ, sin 2ϕ, ..., sin 6ϕ
Computes H_n^m (see fig. 2)
Computes C_{rn}, $C_{\phi n}$ with equations 8, 9
Computes K_n^m (see fig. 3)
Computes $C_{\Theta n}$ with equation 7
Computes H_r, H_Θ, H_ϕ from equation 4, 5, 6
Computes M from equation 23

Figure 1, computation of H_r, H_Θ, H_ϕ, M.

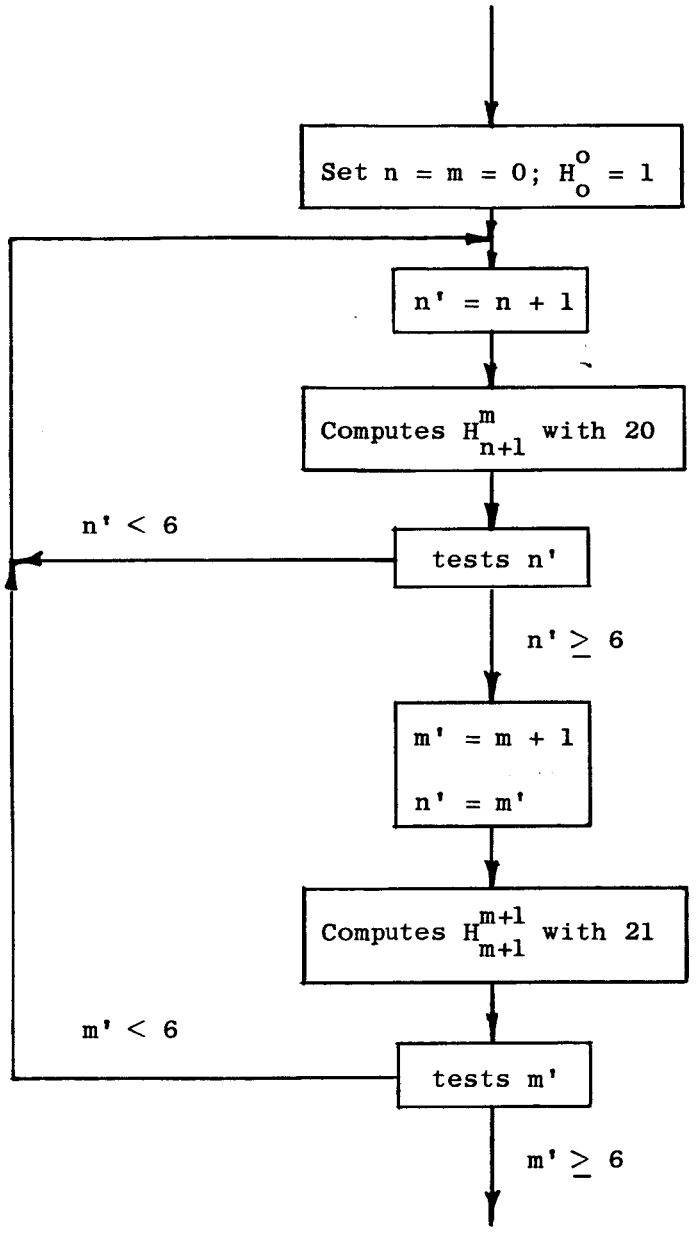


Figure 2, computation of H_o^o, $H_1^o, \ldots, H_6^o$, H_1^1, $H_2^1 \ldots H_6^1$, $H_2^2 \ldots H_6^2, \ldots\ldots H_6^5$, H_6^6

in the given sequence.

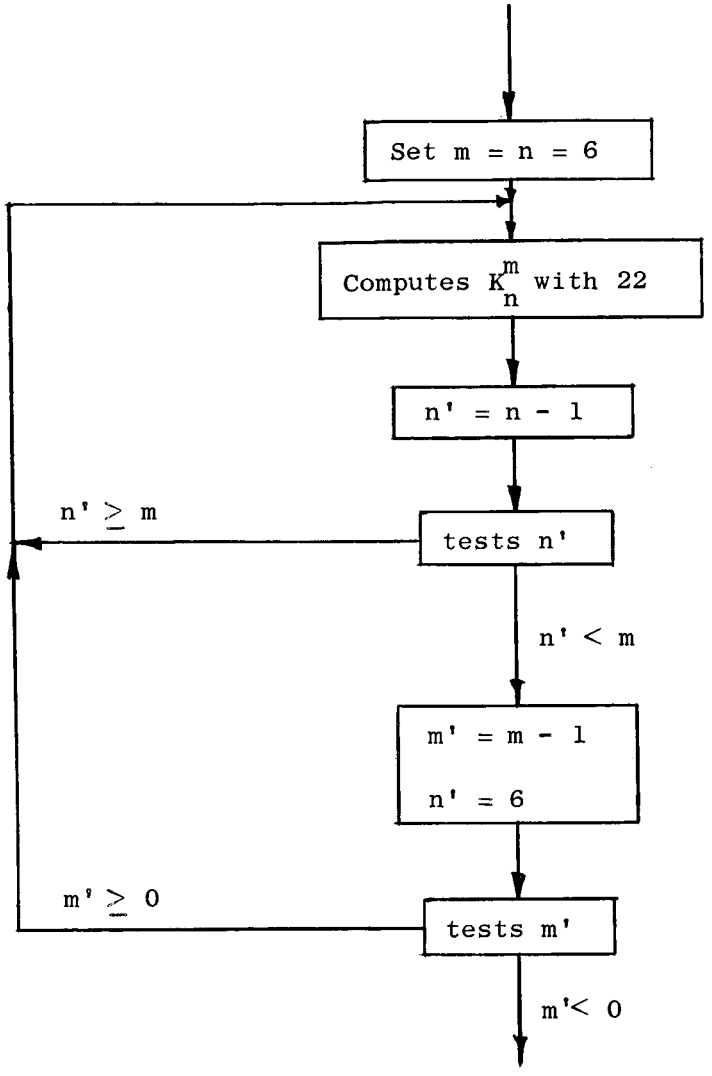
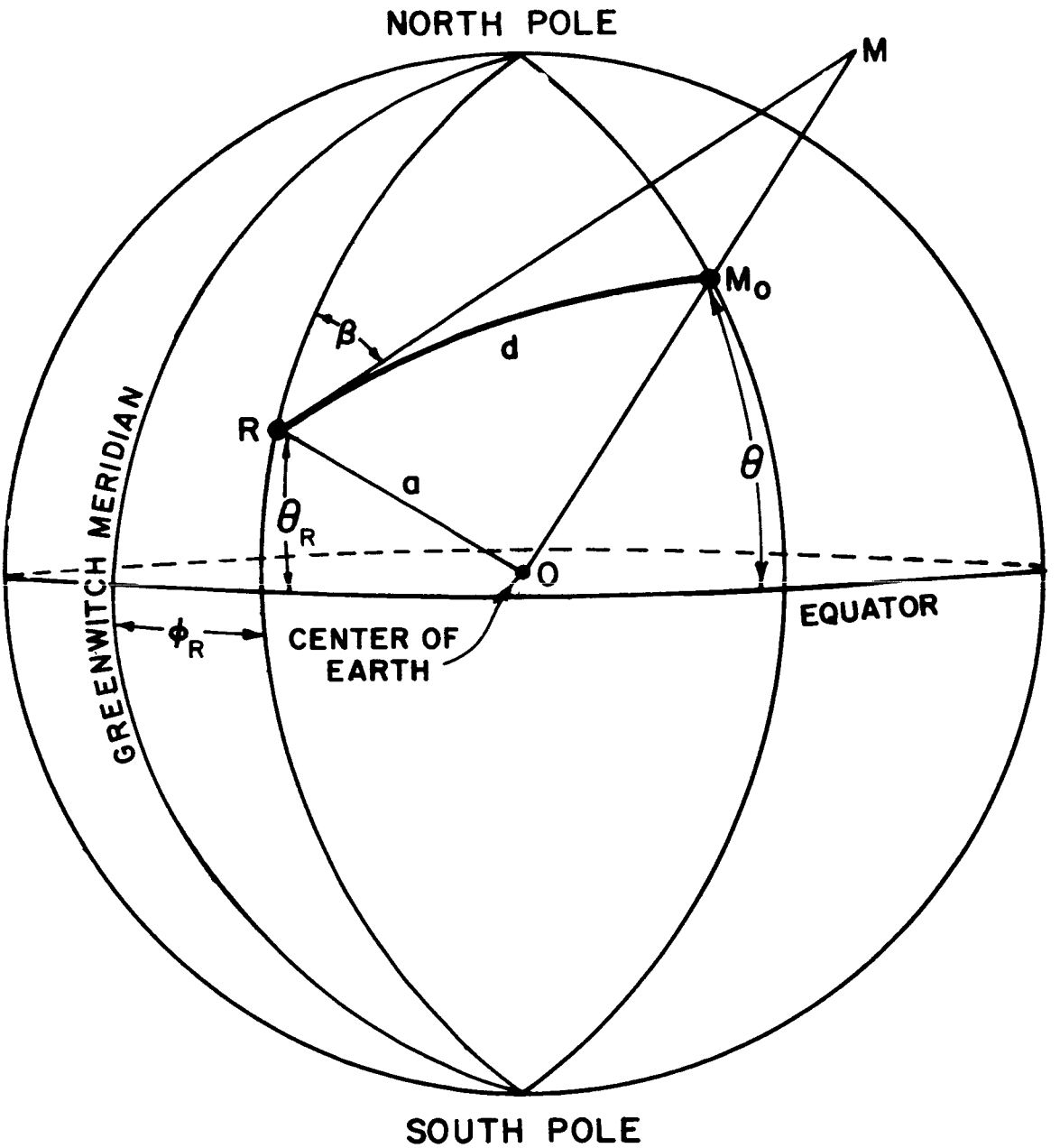


Figure 3, computation of K_6^6, K_6^5, ...K_6^2, ...K_2^2, K_6^1....K_2^1, K_1^1, K_o^o in the given sequence.



R: RECEIVING STATION GIVEN BY a, θ_R, ϕ_R

a: EARTH RADII (6371.2 km)

M: POINT OF MAGNETIC FIELD COMPUTATION GIVEN BY ϵ, β, h (SEE FIGURES 5 & 6)

M_o: SUBFIELD POINT GIVEN BY β, d

Figure 4. Geometry Illustrating Different Parameters.

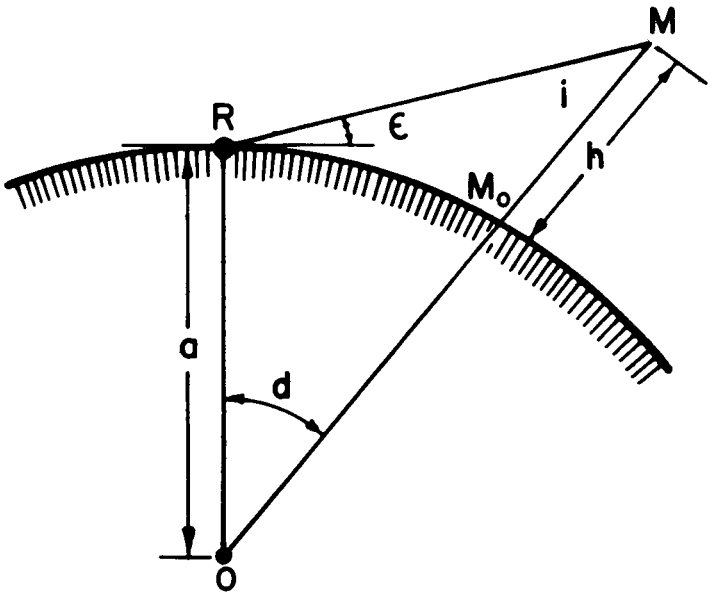


Figure 5. Plane View

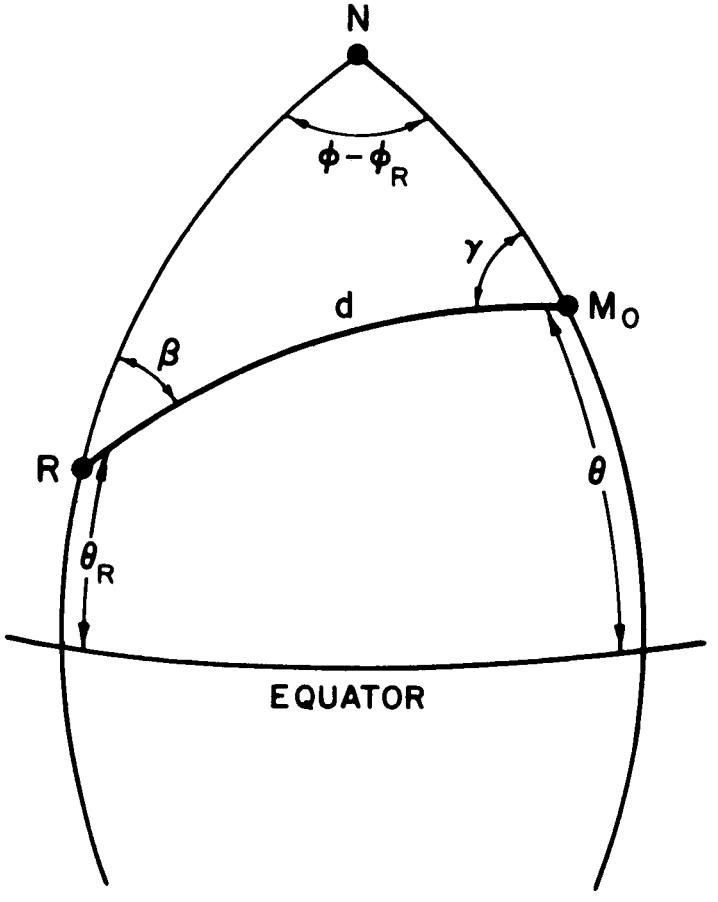


Figure 6. Spherical Triangle

$$\sin i = \frac{a}{a + h} \cos \epsilon \tag{11}$$

$$\cos i = \sqrt{1 - \sin^2 i} \tag{12}$$

$$d = \pi/2 - \epsilon - i \tag{13}$$

$$\cos \theta = \sin \theta_R \cos d + \cos \theta_R \sin d \cos \beta \tag{14}$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \tag{15}$$

$$\sin \gamma = \frac{\sin \beta}{\sin \theta} \cos \theta_R \tag{16}$$

$$\cos \gamma = \frac{\sin \theta_R - \cos d \cos \theta}{\sin d \sin \theta} \tag{17}$$

$$\sin (\phi - \phi_R) = \frac{\sin \beta}{\sin \theta} \sin d \tag{18}$$

$$H_o^o = 1 \tag{19}$$

$$H_{n+1}^m = \mu H_n^m - \frac{n^2 - m^2}{4n^2 - 1} H_{n-1}^m \tag{20}$$

$$H_{n+1}^{m+1} = \sqrt{1 - \mu^2} H_n^m \quad n = m \tag{21}$$

$$K_n^m = - n \mu H_n^m + \frac{n^2 - m^2}{2n - 1} H_{n-1}^m \tag{22}$$

$$M = - H_r - H_\theta \cos \gamma \tan i - H_\phi \sin \gamma \tan i \tag{23}$$

4. Computer Notation

The symbology used hereafter is that of the form acceptable to a computer in compiler language. This means that superscripts and subscripts are not allowable, and summation indexes must go from 1 to n+1 instead of 0 to n. This is because the information is stored in the computer in matrix form and rows and columns of computer matrices begin with one in FORTRAN.

The following symbols are thus used in place of those in equations four through twenty-three:

$$A = a = 6371.2 \text{ km}$$

$$GG(m,n) = g_n^m$$

$$HH(m,n) = h_n^m$$

$$H(m,n) = H_n^m$$

$$F(m,n) = K_n^m$$

$$CR(N) = C_{rn}$$

$$CTH(N) = C_{\theta n}$$

$$CPH(N) = C_{\phi n}$$

$$SINTHE = \sin \theta$$

$$COSTHE = \cos \theta$$

$$SINAI = \sin i$$

$$COSAI = \cos i$$

$$AI = i$$

$$SINGAM = \sin \gamma$$

$$\text{COSGAM} = \cos \gamma$$

$$\text{SINTR} = \sin (\phi - \phi_R)$$

$$\text{APHI} = \phi$$

$$\text{SINP(N)} = \sin n \phi$$

$$\text{COSP(N)} = \cos n \phi$$

And it is to be noted that since the index of a matrix element must be one, then the following notation occurs:

$$\text{SINP(1)} = \text{SIN } 0 \phi$$

$$= 0$$

that is: $$N = n + 1$$

The following symbols are used in the program as intermediate variables:

$$\text{X(M,N)} = \text{partial sum of } C_{rn}$$

$$\text{Y(M,N)} = \text{partial sum of } C_{\phi n}$$

$$\text{Z(M,N)} = \text{partial sum of } C_{\theta n}$$

$$\text{FIN 1} = \text{partial sum of } H_r$$

$$\text{FIN 2} = \text{partial sum of } H_\theta$$

$$\text{FIN 3} = \text{partial sum of } H_\phi$$

Finally, the inputs and outputs are:

S = north latitude of observer, radians floating = θ_R

T = east longitude of observer, radians floating = ϕ_R

U = azimuth of magnetic field point, radians floating = β

V = elevation of magnetic field point, radians floating = ϵ

VLTH = height of magnetic field point, km = h

GG = Gaussian coefficients

HH = Gaussian coefficients

HR = upward component of magnetic field, MKS units

HTHETA = southward component of magnetic field, MKS units

HPHI = eastward component of magnetic field, MKS units

FLDM = magnitude of the magnetic field, MKS units

5. Gaussian Coefficients

The gaussian coefficients used in computing the magnetic field are those of Jensen and Cain (1962). The coefficients, in MKS units, are reproduced in table one.

Table 1. Gaussian Coefficients in MKS Units

GG(m,n)

	1	2	3	4	5	6	7
1							
2	24.20046	1.70885					
3	1.91264	-4.07858	-1.06483				
4	-2.50772	4.94415	-1.98132	-0.51694			
5	-3.32586	-3.60470	-1.73439	0.55768	-0.16266		
6	1.29361	-2.73802	-1.54754	-0.04838	0.22083	0.05547	
7	-1.55359	-0.38619	0.25560	1.70399	0.08364	0.01806	0.08873

HH(m,n)

	1	2	3	4	5	6	7
1							
2		-4.61064					
3		2.63592	-0.12565				
4		1.18332	-0.32438	0.01671			
5		-0.94100	0.79625	0.03422	0.11021		
6		-0.06334	-0.15915	0.36582	0.19266	-0.09693	
7		-0.45821	-0.69511	-0.27104	-0.00939	-0.08881	-0.02586

6. Conclusion

Because the program is of a general nature, it can be incorporated as a subroutine into any program requiring knowledge of the earth's magnetic field. For example, the use of the subroutine has resulted in a saving of many man-hours and improved accuracy in Faraday rotation studies, at a rather small cost, as compared to the older M-chart method. For any application where a spherical harmonic expansion of the earth's magnetic field is desired, this program can be utilized.

```
      SUBROUTINE MAGFLD (S,T,U,V,VLTH,GG,HH,HR,HTHETA,HPHI,FLDM)          MAGFLD
      DIMENSION H(7,7),F(7,7),GG(7,7),HH(7,7),X(7,7),Y(7,7),Z(7,7),       MAGFLD
     1SINP(7),COSP(7),CR(7),CPH(7),CTH(7)                                 MAGFLD
      SINAI = (6371.2/(6371.2 + VLTH))*COS (V)                            MAGFLD
      COSAI = SQRT (ABSF (1.0 - SINAI*SINAI))                             MAGFLD
      TANAI = SINAI/COSAI                                                 MAGFLD
      AI = ATAN (TANAI)                                                   MAGFLD
      D = 1.57079633 - V - AI                                             MAGFLD
      COSTHE = SIN (S)*COS (D) + COS (S)*SIN (D)*COS (U)                  MAGFLD
      SINTHE =SQRT (ABSF (1.0 - COSTHE*COSTHE))                           MAGFLD
      SINGAM = SIN (U)*COS (S)/SINTHE                                     MAGFLD
      COSGAM = (SIN (S) - COS (D)*COSTHE)/(SIN (D)*SINTHE)                MAGFLD
      SINTR = SIN (U)*SIN (D)/SINTHE                                      MAGFLD
      APHI = ATAN (SINTR/SQRT (ABSF (1.0 - SINTR*SINTR))) + T             MAGFLD
      SINP(1) = 0.0                                                       MAGFLD
      COSP(1) = 1.0                                                       MAGFLD
      DO 30 N = 1,6                                                       MAGFLD
      SINP(N+1) = SIN (FLOATF(N)*APHI)                                    MAGFLD
30    COSP(N+1) = COS (FLOATF(N)*APHI)                                    MAGFLD
      SUM1 = 0.0                                                          MAGFLD
      SUM2 = 0.0                                                          MAGFLD
      SUM3 = 0.0                                                          MAGFLD
      DO 4 N =1,7                                                         MAGFLD
      DO 5 M =1,7                                                         MAGFLD
      X(M,N) = 0.0                                                        MAGFLD
      Y(M,N) = 0.0                                                        MAGFLD
      Z(M,N) = 0.0                                                        MAGFLD
      F(M,N) = 0.0                                                        MAGFLD
5     H(M,N) = 0.0                                                        MAGFLD
4     CONTINUE                                                            MAGFLD
      H(1,1) = 1.0                                                        MAGFLD
      M = 1                                                               MAGFLD
      N = 1                                                               MAGFLD
21    N = N + 1                                                           MAGFLD
      IF (N - 2)1,1,2                                                     MAGFLD
1     H(M,N) = COSTHE*H(M,N-1)                                            MAGFLD
      GO TO 3                                                             MAGFLD
2     H(M,N) = COSTHE*H(M,N-1) - (FLOATF ((N-2)*(N-2) - (M-1)*(M-1))/FLOMAGFLD
     1ATF (4*(N-2)*(N-2) - 1))*H(M,N-2)                                   MAGFLD
3     IF (N - 7)21,22,22                                                  MAGFLD
22    M = M + 1                                                           MAGFLD
      N = M                                                               MAGFLD
      H(M,N) = SINTHE*H(M-1,N-1)                                          MAGFLD
      IF (M - 7)21,23,23                                                  MAGFLD
23    DO 7 N = 1,7                                                        MAGFLD
      DO 6 M = 1,N                                                        MAGFLD
      X(M,N) = H(M,N)*(GG(M,N)*COSP(M) + HH(M,N)*SINP(M))                 MAGFLD
6     SUM1 = SUM1 + X(M,N)                                                MAGFLD
      CR(N) = SUM1                                                        MAGFLD
7     SUM1 = 0.0                                                          MAGFLD
      DO 9 N = 1,7                                                        MAGFLD
      DO 8 M = 1,N                                                        MAGFLD
      Y(M,N) = FLOATF (M-1)*H(M,N)*(-1.0*GG(M,N)*SINP(M) + HH(M,N)*COSP(MAGFLD
     1M))                                                                 MAGFLD
8     SUM2 = SUM2 + Y(M,N)                                                MAGFLD
      CPH(N) = SUM2                                                       MAGFLD
9     SUM2 = 0.0                                                          MAGFLD
      M = 7                                                               MAGFLD
      N = 7                                                               MAGFLD
11    IF (N - 1)12,12,13                                                  MAGFLD
```

```
12    F(M,N) = -1.0*FLOATF (N-1)*COSTHE*H(M,N)                                  MAGFLD
      GO TO 14                                                                  MAGFLD
13    F(M,N) = -1.0*FLOATF (N-1)*COSTHE*H(M,N) + (FLOATF ((N-1)*(N-1) -         MAGFLD
     1(M-1)*(M-1))/FLOATF (2*(N-1) - 1))*H(M,N-1)                               MAGFLD
14    N = N - 1                                                                 MAGFLD
      IF (N - M)10,11,11                                                        MAGFLD
10    M = M - 1                                                                 MAGFLD
      N = 7                                                                     MAGFLD
      IF (M)15,11,11                                                            MAGFLD
15    DO 17 N =1,7                                                              MAGFLD
      DO 16 M = 1,N                                                             MAGFLD
      Z(M,N) = F(M,N)*(GG(M,N)*COSP(M) + HH(M,N)*SINP(M))                       MAGFLD
16    SUM3 = SUM3 + Z(M,N)                                                      MAGFLD
      CTH(N) = SUM3                                                             MAGFLD
17    SUM3 = 0.0                                                                MAGFLD
      FIN1 = 0.0                                                                MAGFLD
      FIN2 = 0.0                                                                MAGFLD
      FIN3 = 0.0                                                                MAGFLD
      DO 18 N = 1,7                                                             MAGFLD
      CNST1 = FLOATF (N)                                                        MAGFLD
      CNST2 = (6371.2/(6371.2 + VLTH))**(CNST1 + 1.0)                           MAGFLD
      PAR1 = CNST1*CNST2*CR(N)                                                  MAGFLD
      FIN1 = FIN1 + PAR1                                                        MAGFLD
      PAR2 = CNST2*CTH(N)                                                       MAGFLD
      FIN2 = FIN2 + PAR2                                                        MAGFLD
      PAR3 = CNST2*CPH(N)                                                       MAGFLD
18    FIN3 = FIN3 + PAR3                                                        MAGFLD
      HR = -1.0*FIN1                                                            MAGFLD
      HTHETA = -1.0*(1.0/SINTHE)*FIN2                                           MAGFLD
      HPHI = (1.0/SINTHE)*FIN3                                                  MAGFLD
      FLDM = -1.0*HR - HTHETA*COSGAM*TANAI - HPHI*SINGAM*TANAI                  MAGFLD
      RETURN                                                                    MAGFLD
      END                                                                       MAGFLD
```

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